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Efficient Estimation in Poststratification under Optimal and Non-optimal Conditions

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SUMMARY

Employing the customary predictive format, as alluded to by Basu (1971), Smith (1976) and several others, for estimation of the population total or the population mean under a fixed population set-up, we have generated a sequence of efficient unbiased poststratification-based estimators. The proposed sequence of estimators is found, under optimal and non-optimal conditions, to be more efficient than the customary poststratified estimator and the usual simple mean. The performance of the proposed sequence of estimators has been examined from the point of view of conditional randomization inference.

Key words: Poststratification, Conditional randomization inference, Non-optimal better estimators.

1. INTRODUCTION

To ascertain whether or not the traditional estimators conform to a certain intuitive in-built feature, Basu (1971), Smith (1976) and several others have pleaded for a plausible predictive format under a fixed-population set-up. Agrawal and Sthapit (1997) have tapped this format to arrive at a sequence of efficient ratio-based and product-based estimators. Agrawal and Panda (1993) have suggested a suitably weighted combination of the customary poststratified estimator ($\bar{y}_{ps}$, say) and simple mean ($\bar{y}$, say) which performs better than either of the estimators $\bar{y}_{ps}$ and $\bar{y}$. In this paper, we have invoked a plausible predictive format under poststratified sampling and have used $\bar{y}_{ps}$ as an input predictor for the non-surveyed part of the population in a repetitive manner, thus obtaining a sequence of poststratification-based estimators.

Agrawal and Panda (1995) proposed a poststratified estimator through use of optimum weights. Here, in this paper, we consider, apart from the optimum situation, a decomposition of optimum weights into non-optimal sub-weights with a view to retaining the superiority of the suggested poststratified estimator over the customary poststratifeid estimator and simple mean.

2. POSTSTRATIFICATION-BASED ESTIMATION AND THE RELATED PERFORMANCE

Consider a population of size N stratified into k strata, the size of the i^{th} stratum being N_i such that $\sum_{i=1}^{k} N_i = N$. A simple random sample of size n is drawn from the population and the sample units are then assigned to the k strata. Suppose that n_i (i = 1, 2, · · · , k) is the number of units that fall into the i^{th} stratum such that $\sum_{i=1}^{k} n_i = n$, n_i varying from sample to sample. Assuming the probability of n_i being zero to be small, the usual unbiased estimator of the population mean $\bar{Y}$ in poststratified sampling is given by

$$\bar{y}_{ps} = \sum_{i=1}^{k} W_i \bar{y}_i$$

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where *ps* stands for poststratification, $W_i = N_i/N$ and $\bar{y}_i$ is the mean of the n_i sample units that fall into stratum $i\,(i = 1, 2, \cdots, k)$. With a view to arriving at a predictive format under the fixed population set-up, we express the population total Y as

$$Y = \sum_{i=1}^{k} \sum_{j \in s_i} y_{ij} + \sum_{i=1}^{k} \sum_{j \in \bar{s}_i} y_{ij} = \sum_{i=1}^{k} n_i \bar{y}_i + \sum_{l \in \bar{s}} y_l \qquad (2.1)$$

where s_i denotes the sample of size n_i selected from the i^{th} stratum and $\bar{s}_i$ is its complement, and $\bar{s}$ is the complement of overall sample $s = \bigcup_{i=1}^{k} s_i$. It is clear from (2.1) that, to estimate the total Y, we have to predict $y_l\,(l \in \bar{s})$ because the first component on the right hand side of (2.1) is known. This is tantamount to stating

$$\hat{Y} = \sum_{i=1}^{k} n_i \bar{y}_i + \sum_{l \in \bar{s}} \hat{y}_l \qquad (2.2)$$

where $\hat{y}_l$ is the implied predictor of $y_l\,(l \in \bar{s})$. Invoking the customary post-stratified estimator $\bar{y}_{ps}$ as an intuitive predictor of y_l in (2.2), we obtain

$$\hat{Y} = \sum_{i=1}^{k} n_i \bar{y}_i + (N-n)\bar{y}_{ps}$$

or say

$$\hat{Y} = \sum_{i=1}^{k} \frac{n_i \bar{y}_i}{N} + \frac{N-n}{N}\bar{y}_{ps} = y_{ps}^{(1)}$$

In the next step, we utilize $\bar{y}_{ps}^{(1)}$ as an intuitive predictor of y_l in (2.2) and this leads to $\bar{y}_{ps}^{(2)}$ given by

$$\bar{y}_{ps}^{(2)} = (1-\lambda^2)y + \lambda^2 \bar{y}_{ps}$$

where $\lambda = 1 - \dfrac{n}{N}$. Repetition of this process r times will culminate in

$$\bar{y}_{ps}^{(r)} = (1-\lambda^r)\bar{y} + \lambda^r \bar{y}_{ps} \qquad (2.3)$$

Having obtained $\bar{y}_{ps}^{(r)}$ given by (2.3), we may, in fact, extend the scope of r to cover negative integer values or even all real values without causing any problem. However, we hereafter consider $r \geq 0$. Such an estimator $\bar{y}_{ps}^{(r)}$, which is unbiased for population mean $\bar{Y}$, will be called poststratification-based estimator of order r. It may be noted that, for $r = 0$, $\bar{y}_{ps}^{(r)}$ (*i.e*, $\bar{y}_{ps}^{(0)}) = \bar{y}_{ps}$ and, as $r \to \infty, y_{ps}^{(r)} \to \bar{y}$. Now, noting that

$$V(\bar{y}) = \frac{\lambda}{n} S^2$$

$$V(\bar{y}_{ps}) = \frac{\lambda}{n} \sum W_i S_i^2 + \frac{N(N-1)\lambda}{n^2} \sum_{i=1}^{k} (1-W_i) S_i^2$$

and

$$\text{Cov}(\bar{y}, \bar{y}_{ps}) = \text{Cov}\left(\sum_{i=1}^{k} \frac{n_i \bar{y}_i}{n}, \sum_{i=1}^{k} W_i \bar{y}_i \right)$$

$$= \frac{\lambda}{n} \sum_{i=1}^{k} W_i S_i^2$$

the variance of $\bar{y}_{ps}^{(r)}$ can be expressed as

$$V(\bar{y}_{ps}^{(r)}) \approx \frac{\lambda}{n} S^2 + \frac{\lambda^{r+1}}{n} Q(\lambda^r - 2)$$
$$+ \frac{\lambda^{2r+1}}{n^2} \left[\frac{N}{N-1} \sum_{i=1}^{k} (1-W_i) S_i^2 \right] \qquad (2.4)$$

where $Q = S^2 - \sum_{i=1}^{k} W_i S_i^2$ and S^2 and S_i^2 are, respectively, the population mean square and the mean square for the i^{th} stratum. The optimum value of r which minimizes the variance expression given in (2.4) is obtainable from

$$\lambda^{r*} = \frac{Q}{R} \qquad (2.5)$$

where $r*$ denotes the optimum value of r, $R = Q + Q_1$ and

$$Q_1 = \frac{1}{f(N-1)}\sum_{i=1}^{k}(1-W_i)S_i^2$$

Barring some exceptional sampling situations, the value of Q will be positive. Henceforth, we will assume $Q > 0$. It can be easily verified that the use of (2.5) will reduce (2.4) to

$$V_{\min}(\bar{y}_{ps}^{(r)}) = V(\bar{y}_{ps}^{(r^*)}) = \frac{\lambda}{n}S^2 - \frac{\lambda Q^2}{nR}$$

which will be smaller that $V(\bar{y}_{ps})$ or $V(\bar{y})$. Rewriting the variance of $\bar{y}_{ps}$ as

$$V(\bar{y}_{ps}) = \frac{\lambda}{n}\sum_{i=1}^{k}W_iS_i^2 + \frac{\lambda}{n}Q_1 = \frac{\lambda}{n}S^2 + \frac{\lambda}{n}(Q_1 - Q)$$

we note that poststratification will be resorted to only when $\frac{Q}{Q_1} > 1$, for otherwise, the simple mean will score over the poststratified estimator $\bar{y}_{ps}$.

Since determination of r^* via (2.5) will not be easy in view of involvement of the population quantities, we discuss non-optimal efficient solution in order to ensure superior performance of $\bar{y}_{ps}^{(r)}$ for values of r other than r^*. The following inequality, obtained from the comparison of relevant variances of $\bar{y}_{ps}^{(r)}$, $\bar{y}_{ps}$ and $\bar{y}$ given above, will render $\bar{y}_{ps}^{(r)}$ more efficient compared to either of $\bar{y}_{ps}$ and $\bar{y}$

$$\frac{\tau-1}{\tau+1} < \lambda^r < \frac{2\tau}{\tau+1} \quad \text{where } \tau = \frac{Q}{Q_1} \tag{2.6}$$

An idea about τ can be had from a pilot or past survey, thus enabling us to decide on τ.

For different values of f and τ, we have used (2.6) to prepare Table 1 which displays bounds on r for which $\bar{y}_{ps}^{(r)}$ performs better than $\bar{y}$ and $\bar{y}_{ps}$.

Table 1. Range of r for different τ and f values

$\tau \backslash f$	.01	.05	.10	.20
0.1	$r > 169.62$	$r > 33.23$	$r > 16.18$	$r > 7.64$
0.5	$r > 40.34$	$r > 7.9$	$r > 3.84$	$r > 1.82$
0.9	$r > 5.38$	$r > 1.05$	$r > .51$	$r > .24$
1	$r > 0$	$r > 0$	$r > 0$	$r > 0$
2	$0 < r < 109.31$	$0 < r < 21.42$	$0 < r < 10.43$	$0 < r < 4.92$
5	$0 < r < 40.34$	$0 < r < 7.90$	$0 < r < 3.85$	$0 < r < 1.82$
10	$0 < r < 19.96$	$0 < r < 3.91$	$0 < r < 1.90$	$0 < r < .90$

To appreciate mathematically the significance of τ which is a pivotal quantity, we suppose $S_i^2 = S_W^2$ which implies that proportional allocation is optimal in the Neyman sense. Then, for large N_i ($i = 1, 2, \cdots, k$)

$$Q = S^2 - \sum_{i=1}^{k}W_iS_i^2 = \sum_{i=1}^{k}W_i(\bar{Y}_i - \bar{Y})^2$$

$$= S_b^2 \cdot \frac{k-1}{N}$$

where $S_b^2 = \frac{1}{k-1}\sum_{i}^{k}N_i(\bar{Y}_i - \bar{Y})^2$

and $Q_1 = \frac{1}{f(N-1)}\sum_{i=1}^{k}(1-W_i)S_i^2 = \frac{(k-1)S_W^2}{f(N-1)}$

and thus $\tau = f.\ F$ where $F = \frac{S_b^2}{S_W^2}$ which is a ratio of 'between' mean squares to 'within' mean squares and is obtained from ANOVA table.

3. PERFORMANCE-SENSITIVITY OF THE PROPOSED ESTIMATOR DUE TO NON-OPTIMALLY OF *r*

We would like to determine the loss in efficiency of $\bar{y}_{ps}^{(r)}$ arising from the use of values of r other than optimum r (i.e. r^*) value. To evaluate this loss, we define a quantity P_I which is the proportional inflation in variance of $\bar{y}_{ps}^{(r)}$ resulting from lack of knowledge of r^* as

$$P_I = \frac{V(\bar{y}_{ps}^{(r)}) - V(\bar{y}_{ps}^{(r^*)})}{V(\bar{y}_{ps}^{(r^*)})} \tag{3.1}$$

After some algebra, P_{I} can be expressed as

$$P_{\mathrm{I}} = \left(\frac{\lambda^{r} - \lambda^{r*}}{1-\lambda^{r*}}\right)^{2} G$$

where $G = \dfrac{V(\bar{y}_{ps}^{(r)}) - V(\bar{y}_{ps}^{(r*)})}{V(\bar{y}_{ps}^{(r)})}$, indicating the gain in efficiency of $\bar{y}_{ps}^{(r)}$ (using r^*) relative to $\bar{y}_{ps}$. The estimator $\bar{y}_{ps}^{(r)}$ will continue to fare better than $\bar{y}_{ps}$ provided

$$P_{\mathrm{I}} < G \Rightarrow \left|\frac{\lambda^{r} - \lambda^{r^*}}{1-\lambda^{r*}}\right| < 1 \Rightarrow 2\lambda^{r^*} - 1 < \lambda^{r} < 1 \quad (3.2)$$

implying thereby that $\bar{y}_{ps}^{(r)}$ will always (irrespective of choice of r) be more efficient than $\bar{y}_{ps}$ if $\lambda^{r*} < \frac{1}{2} (\Rightarrow \tau < 1)$. But, for $\lambda^{r^*} > \frac{1}{2} (\Rightarrow \tau > 1)$ we can manipulate (3.2) to obtain

$$r < \frac{\log(2\lambda^{r*} - 1)}{\log \lambda} \square \frac{2Q_1}{f\,Q} = \frac{2}{f\tau} \quad (3.3)$$

Now, turning to the case involving $\bar{y}$, we can express $P_{\mathrm{I}} = \delta^{2} \cdot G'$ where $G' = \dfrac{V(\bar{y}) - V(y_{ps}^{(r^*)})}{V(\bar{y}_{ps}^{(r*)})}$ and $\lambda^{r} = (1+\delta)\lambda^{r^*}$, G' indicating the gain in efficiency of $\bar{y}_{ps}^{(r)}$ (using r^*) relative to $\bar{y}$. The estimator $\bar{y}_{ps}^{r}$ (for a non-optimal r) will be more efficient than $\bar{y}$ provided

$$P_{\mathrm{I}} < G' \Rightarrow |\delta| < 1 \Rightarrow \lambda^{r} < 2\lambda^{r^*} \Rightarrow \tau > \frac{\lambda^{r}}{2-\lambda^{r}} \quad (3.4)$$

Alternatively, $\bar{y}_{ps}^{(r)}$ will fare better than $\bar{y}$ if

$$r > \frac{1}{f\tau} - \frac{\ln 2}{f} \quad (3.5)$$

where $\tau > 1$. Combining (3.3) and (3.5), we conclude that, for $\bar{y}_{ps}^{(r)}$ to perform better than $\bar{y}$ and $\bar{y}_{ps}$, we have (for $\tau > 1$)

$$\frac{1}{f\tau} - \frac{\ln 2}{f} < r < \frac{2}{f\tau} \quad (3.6)$$

Note also that, if $\dfrac{\lambda^{r}}{2-\lambda^{r}} < \tau < 1$ (for and r), $\bar{y}_{ps}^{(r)}$ will be superior to $\bar{y}$ and $\bar{y}_{ps}$. It can be verified from (3.4) that, if $\tau < 1$, the values of r that render $\bar{y}_{ps}^{(r)}$ more efficient than $\bar{y}$ are given by

$$r > \frac{\tau - \ln(2\tau)}{f} \quad (3.7)$$

Alternatively, taking $r = (1+\delta')r^*$ where δ' is the proportional deviation in r^*, we can express

$$P_{\mathrm{I}} = \left\{\left(\frac{Q}{R}\right)^{\delta'} - 1\right\}^{2} \quad G' = \frac{\delta'^{2}}{r^{2}} G' \text{ if } \tau > 1$$

Numerical Illustration

Example: The following data have been taken from Sarndal *et al.* (1992, p.119)

Stratum i	N_i	$\sum_{j=1}^{N_i} y_{ij}$	$\sum_{j=1}^{N_i} y_{ij}^{2}$
1	105	1098.9	21855.05
2	19	3445.9	1822736.83

(i) For n = 30, f = 0.242, λ = 0.758, we have $\lambda^{r*} = 0.64 \Rightarrow r^* = 1.6$ and $\tau = \dfrac{\lambda^{r*}}{1-\lambda^{r*}} = 1.775$.

Since $\tau > 1$, it is clear from (3.6) that, for $\bar{y}_{ps}^{(r)}$ to be more efficient than $\bar{y}$ and $\bar{y}_{ps}$, we should have $0 < r < 4.66$.

(ii) For n = 15, f = 0.121, λ = 0.897, we have λ^{r^*} = .47, r^* = 0.5.851 and τ = 0.887.

As $\tau < 1$, we conclude from (3.2) that $\bar{y}_{ps}^{(r)}$ (whatever be r) will always be better than $\bar{y}_{ps}$. However, for $\bar{y}_{ps}^{(r)}$ to perform better than $\bar{y}$, we get $r > 2.61$ from (3.7). Thus a choice of $r > 2.61$ will ensure superiority of $\bar{y}_{ps}^{(r)}$ vis-a-vis $\bar{y}_{ps}$ and $\bar{y}$.

For the above example with $n = 30$, Table 2 presents appraisal of the impact of departure from r^* in terms of loss in the efficiency of $\bar{y}_{ps}^{(r)}$ as a result of employing some non-optimum r instead of r^*.

Table 2. Loss in efficiency of $\bar{y}_{ps}^{(r)}$ as a result of departure from r^*

δ′	P_{I}
0.05	0.000091
0.10	0.000357
0.15	0.000786
0.20	0.001367
0.25	0.002090
0.30	0.002944
0.35	0.003921
0.40	0.005012
0.45	0.006208
0.50	0.007500

Table 2 clearly reflects that proportional deviations to the extent of 50% from r^* cause only .75% proportional inflation on $V(\bar{y}_{ps}^{(r)})$ relative to $V(\bar{y}_{ps}^{(r^*)})$. In other words, there is insignificant or no loss in efficiency of $\bar{y}_{ps}^{(r)}$ when we conceive departures from r^*, at least to the extent envisaged in the above table.

4. CONDITIONAL RANDOMIZATION INFERENCE

Following the work of authors such as Holt and Smith (1979), Smith (1991), Valliant (1993), Agrawal and Panda (1995) with regard to the use of conditional inference in poststratification, we would now like to examine the performance of $\bar{y}_{ps}^{(r_c)}$ in the conditional case by conditioning the mean square error on actual sample size from different strata and the same is then expressible as

$$\text{MSE}(\bar{y}_{ps}^{(rc)} \mid n) = V(\bar{y}_{ps}^{(r_c)} \mid n) + \{Bias(\bar{y}_{ps}^{(r_c)} \mid n)\}^2$$

$$= \sum_{i=1}^{k} \psi_i \left(\frac{n_i}{n} - \lambda^{rc} k_i \right)^2 \mid (1-\lambda^{rc})^2 \left(\sum_{i=1}^{k} K_i \bar{Y}_i \right)^2 \quad (4.1)$$

where $W_i = \dfrac{N_i}{N}, K_i = \dfrac{n_i}{n} - \dfrac{N_i}{N}$ and $\psi_i = \left(\dfrac{1}{n_i} - \dfrac{1}{N_i} \right) S_i^2$

The subscript c' in the above discussion indicates 'conditional' case. Using the optimal value of r_c, say, r_c^* we can find

$$\text{MSE}\left(y_{ps}^{(r_c^*)} \mid n\right) = \sum_{i=1}^{k} \psi \left(\frac{n_i}{n} \right)^2 + \left(\sum_{i=1^k} K_i \bar{Y}_i \right)^2 - \frac{\left[\sum_{i=1}^{k} K_i \psi_i \frac{n_i}{n} + \left(\sum_{i=1}^{k} K_i \bar{Y}_i \right)^2 \right]^2}{\sum_{i=1}^{k} K_i^2 \psi_i + \left(\sum_{i=1}^{k} K_i \bar{Y}_I \right)^2} \quad (4.3)$$

which will be smaller that $V(\bar{y}_{ps} \mid n)$ or $\text{MSE}(\bar{y} | n)$ given by

$$V(\bar{y}_{ps} \mid n) = \sum_{i=1}^{k} W_i^2 \psi_i \quad (4.4)$$

$$\text{MSE}(\bar{y} \mid n) = \sum_{i=1}^{k} \left(\frac{n_i}{n} \right)^2 \psi_i + \left(\sum_{i=1}^{k} K_i \bar{Y} i \right)^2 \quad (4.5)$$

5. PERFORMANCE-SENSITIVITY UNDER CONDITIONAL RANDOMIZATION INFERENCE

It is of interest to know that potential loss in efficiency of the proposed post stratified estimator $\bar{y}_{ps}^{(r_c)}$ if we use some λ^{r_c} other than $\lambda_c^{r^*}$. For this purpose, we define, under conditional randomization inference, a measure P_{IC} similar to P_{I}, i.e.,

$$P_{\text{IC}} = \frac{\text{MSE}(y_{ps}^{(r_c)} \mid n) - \text{MSE}(\bar{y}_{ps}^{(r^*c)} | n)}{\text{MSE}(y_{ps}^{(r^*c)} | n)}$$

We can, after simplification, express

$$P_{IC} = \left(\frac{\lambda^{r_c} - \lambda^{r_c^*}}{1-\lambda_c^{r^*}}\right) G_c'$$

where $G_c' = \dfrac{\mathrm{V}(\bar{y}_{ps} \mid n) - \mathrm{MSE}(y_{ps}^{(r_c^*)} \mid n)}{MSE(y_{ps}^{(r_c^*)} \mid n)}$

In the case of conditional randomization, $\bar{y}_{ps}^{(r_c)}$ will continue to fare better than y_{ps} provided

$$P_{IC} < G_c' \Rightarrow \left|\frac{\lambda^{r_c} - \lambda^{r_c^*}}{1-\lambda^{r_c^*}}\right| < 1 \Rightarrow r_c < \frac{\log(2\lambda^{r_c^*} - 1)}{\log\lambda} \quad (5.1)$$

Proceeding exactly in the same way as in Section 3, we can show that, in the conditional case, $\bar{y}_{ps}^{(r_c)}$ will perform better than $\bar{y}$ if

$$r_c > \frac{\log(2\lambda^{r_c^*})}{\log\lambda} \quad (5.2)$$

Combining (5.1) and (5.2), we conclude that, in the conditional case, $\bar{y}_{ps}^{(r_c)}$ fares better than $\bar{y}$ and $\bar{y}_{ps}$ if

$$\frac{\log(2\lambda^{r_c^*})}{\log\lambda} < r_c < \frac{\log(2\lambda^{r_c^*} - 1)}{\log\lambda} \quad (5.3)$$

The bounds on r_c given by (5.3) may be termed as 'efficiency bounds'. Taking $r_c = r_c^*(1+\delta_c')$ where δ_c' denotes proportional deviation in r_c^*, we can express

$$P_{IC} = \left\{(\lambda^{r_c^*})^{\delta_c'} - 1\right\}^2 \cdot \frac{MSE(\bar{y} \mid n) - MSE(\bar{y}_{ps}^{(r_c^*)} \mid n)}{MSE(\bar{y}_{ps}^{(r_c^*)} \mid n)}$$

To illustrate the above results relating to $\bar{y}_{ps}^{(r_c)}$ under the framework of conditional randomization inference, we consider the following theoretical numerical example due to Holt and Smith (1979) which shows that there exists a sequence of non-optimal efficient estimators (based on use of some r_c other than r_c^*) ensuring superior performance of $\bar{y}_{ps}^{(r_c)}$ compared to simple mean and traditional poststratified estimator.

Example: A population which is postsratified into two strata has the following characteristics.

$$\bar{Y} = 0,\ S^2 = 2,\ N_1/N = N_2/N = \frac{1}{2}$$

$$S_1^2 = S_2^2 = 1,\ \bar{Y}_1 = -1, \bar{Y}_2 = 1 \text{ and } n = 20$$

However, instead of ignoring finite population correction factors as assumed by Holt and Smith (1979), we retain them by considering $N_1 = N_2 = 100$ and $S_1^2 = S_2^2 = 2$ in respect of the two strata.

From the standpoint of conditional randomization inference, we need, because of reasons of symmetry, to discuss the configurations from $n_1 = 1, n_2 = 19$ to $n_1 = 9$, $n_2 = 11$. We have excluded the case $n_1 = n_2 = 10$ as λ_c^{r*} is not defined in view of K_i becoming zero when $\dfrac{n_i}{n} = \dfrac{N_i}{N}\ (i = 1, 2)$.

To appraise the performance of $\bar{y}_{ps}^{(r)}$ under conditional randomization inference, we have prepared Table 3 which reflects the performance of $\bar{y}_{ps}^{(r_c)}$ vis-a-vis $\bar{y}_{ps}$ and $\bar{y}$ when r_c^* is employed. More importantly, we display possible values of r_c (efficiency bounds of r_c) for which $\bar{y}_{ps}^{(r_c)}$ performs better that $\bar{y}_{ps}$ and $\bar{y}$.

Table 3. A Comparison of $\bar{y}_{ps}^{(r_c)}$, $\bar{y}_{ps}$ and $\bar{y}$ under the given configurations of sample sizes

n_1	r_c^*	MSE $(y_{ps}^{r_c^*} \mid n)$	Efficiency bounds of r_c	V $(\bar{y}_{ps} \mid n)$	MSE $(\bar{y} \mid n)$
1	4.05	0.3683	$0 < r < 11.24$	0.5163	0.8919
2	2.35	0.2288	$0 < r < 5.47$	0.2678	0.7236
3	1.71	0.1702	$0 < r < 3.81$	0.1861	0.5751
4	1.39	0.1386	$0 < r < 3.02$	0.1462	0.4464
5	1.20	0.1194	$0 < r < 2.57$	0.1233	0.3375
6	1.08	0.1070	$0 < r < 2.29$	0.1090	0.2484
7	1.01	0.0985	$0 < r < 2.14$	0.0999	0.1785
8	0.95	0.0938	$0< r < 2.00$	0.0942	0.1296
9	0.92	0.0909	$0 < r < 1.94$	0.0910	0.0999

Table 4. Loss of efficiency of $\bar{y}_{ps}^{(r_c)}$, as a result of departures from r_c^*

n_1	r_c	P_{IC}
1	5.0580	0.014500
2	2.9340	0.007800
3	2.1440	0.004600
4	1.7386	0.002900
5	1.4988	0.001770
6	1.3470	0.001000
7	1.2630	0.000560
8	1.1869	0.000230
9	1.1525	0.000057

Table 4 underscores the fact that, for above example, the values of r_c that embody deviations of 25% from r^*_c cause very little inflation in minimum conditional mean square error of $y_{ps}^{(r_c)}$ as indicated by column P_{IC}. In other words, there is insignificant loss in efficiency of $\bar{y}_{ps}^{(r_c)}$ when we conceive departures from r_c^*.

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